SUB - SME 306 - 14/08/2013 - Nome:........................ N.USP:.........
Importante:
- Explique de maneira clara as soluções, preferencialmente baseando-se no formulário.
** Passe as questões a limpo iniciando na folha de questões. **

1	
2	
3	
Total	

Exercício 1. *Encontre o polinômio de grau* 3 *que interpola* $x^{\sqrt{x}}$ *nos pontos* $x = 1$, $x = 2$, $x = 3$, $x = 4$. *Encontre o polinômio de mesmo grau que minimiza erro quadrático nos mesmos pontos (critério de erro discreto).*

Exercício 2. *Resolva* $\dot{x}(t) = t\exp(-x)$, $x(0) = 1$, *no intervalo* $t \in [0 \quad 1]$ *usando método de Taylor de ordem 2 com espaçamento* $h = 0.2$.

Exercício 3. *Ao aproximar uma função* f *por mínimos quadrados contínuo no intervalo* $[-1, 1]$, *obteve-se* $P_f(x) = 0,1\phi_0 + 0,5\phi_1 + 2\phi_2 + 1,2\phi_3$. *Para uma outra função* g, *temos* $P_g(x) = 1\phi_0 + 2\phi_1 + 0,5\phi_2$. *Encontre o polinômio que melhor aproxima* $h = f + 2g + 5$, *de grau* 2, *na forma* $P_h(x) = a_0 + a_1x + a_2x^2$ *(isto é, encontre os valores de* a_0, a_1, a_2*).*

Solução da questão 1)

FORMULÁRIO: $L_{n,k}(x)=\dfrac{(x-x_0)\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_n)}$; $\quad P(x)=\sum_{k=0}^{n} f(x_k)L_{n,k}(x)$; $\quad f(x)=P(x)+\dfrac{f^{(n+1)}(\xi(x))}{(n+1)!}\prod_{k=0}^{n}(x-x_k)$

$$P(x)=\frac{(x-x_j)P_{0,1,\ldots,j-1,j+1,\ldots,n}-(x-x_i)P_{0,1,\ldots,i-1,i+1,\ldots,n}}{(x_i-x_j)};\quad P(x)=\sum_{i=0}^{n}F_{i,i}\prod_{j=0}^{i-1}(x-x_j)$$

$$f(x)=H_{2n+1}(x)+\frac{\prod_{k=0}^{n}(x-x_k)^2}{(2n+2)!}f^{(2n+2)}(\xi(x));\ \ f'(x_j)=\sum_{k=0}^{n}f(x_k)L_k'(x_j)+\frac{f^{(n+1)}(\xi(x_j))}{(n+1)!}\prod_{k\neq j}^{n}(x_j-x_k)$$

$$A=\begin{bmatrix}1 & 0 & 0 & \cdots & \\ h_0 & 2(h_0+h_1) & h_1 & 0 & \cdots\\ 0 & h_1 & 2(h_1+h_2) & h_2 & \\ \vdots & \vdots & \vdots & & \end{bmatrix};\quad B=\begin{bmatrix}0\\ 3(a_2-a_1)/h_1-3(a_1-a_0)/h_0\\ \vdots\\ 3(a_n-a_{n-1})/h_{n-1}-3(a_{n-1}-a_{n-2})/h_{n-2}\\ 0\end{bmatrix};\ a_j=f(x_j);$$

$$b_j=(a_{j+1}-a_j)/h_j-h_j(2c_j+c_{j+1})/3;\ \ d_j=(c_{j+1}-c_j)/3h_j$$

Certas linhas de A e B podem ser: $2h_0,h_0,0,\ldots;\ 0,\ldots,0,h_{n-1},2h_{n-1};\ 3(a_1-a_0)/h_0-3f'(a);\ 3f'(b)-3(a_n-a_{n-1})/h_{n-1}$

$$f'(x_0)=\frac{1}{2h}\left[-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right]+\frac{h^2}{3}f^{(3)}(\xi);\quad f'(x_0)=\frac{1}{2h}\left[f(x_0+h)-f(x_0-h)\right]+\frac{h^2}{6}f^{(3)}(\xi)$$

$$f'(x_0)=\frac{1}{h}\left[f(x_0+h)-f(x_0)\right]+\frac{h}{2}f^{(2)}(\xi);\quad R_{k,j}=R_{k,j-1}+\frac{R_{k,j-1}-R_{k-1,j-1}}{4^{j-1}-1}$$

$$\int_a^b f(x)dx-\sum_{i=0}^{n}a_if(x_i)=\frac{h^{n+3}f^{(n+2)}(\xi)}{(n+2)!}\int_0^n t^2(t-1)\cdots(t-n)dt,\ (n\text{ par})\quad \text{OU}\quad \frac{h^{n+2}f^{(n+1)}(\xi)}{(n+2)!}\int_0^n t(t-1)\cdots(t-n)dt,(n\text{ impar}).$$

$$\int_a^b f(x)dx=\frac{h}{3}\left[f(a)+2\sum_{j=1}^{n/2-1}f(x_{2j})+4\sum_{j=1}^{n/2}f(x_{2j-1})+f(b)\right]-\frac{b-a}{180}h^4f^{(4)}(\mu);\quad e(h)\le nh\varepsilon=(b-a)\varepsilon$$

$$\int_a^b f(x)dx=\frac{h}{2}\left[f(a)+2\sum_{j=1}^{n-1}f(x_j)+f(b)\right]-\frac{b-a}{12}h^2f^{(2)}(\mu);\quad \int_a^b f(x)dx=2h\sum_{j=0}^{n/2}f(x_{2j})+\frac{b-a}{6}h^2f^{(2)}(\mu)$$

$$\left|\int_a^b f(x)dx-S\left(a,\frac{a+b}{2}\right)-S\left(\frac{a+b}{2},b\right)\right|\approx\frac{1}{15}\left|S(a,b)-S\left(a,\frac{a+b}{2}\right)-S\left(\frac{a+b}{2},b\right)\right|$$

$$w_0=\alpha;\ \ w_{i+1}=w_i+hT^{(n)}(t_i,w_i);\ \ T^{(n)}(t_i,w_i)=f(t_i,w_i)+(h/2)f'(t_i,w_i)+\cdots+(h^{n-1}/n!)f^{(n-1)}(t_i,w_i);$$

$$|y''(t)|\le M;\ \ |y(t_i)-e_i|\le\frac{hM}{2L}(e^{L(t_i-a)-1});\ \ w_0=\alpha;\ \ w_{i+1}=w_i+hf\left(t_i+\frac{h}{2},w_i+\frac{h}{2}f(t_i,w_i)\right);$$

$$w_0=\alpha;\ \ w_{i+1}=w_i+(h/2)[f(t_i,w_i)+f(t_i,w_i+hf(t_i,w_i))];$$

$$w_0=\alpha;\ \ k_1=hf(t_i,w_i);\ \ k_2=hf\left(t_i+\frac{h}{2},w_i+\frac{1}{2}k_1\right);\ \ k_3=hf\left(t_i+\frac{h}{2},w_i+\frac{1}{2}k_2\right);\ \ k_4=hf\left(t_{i+1},w_i+k_3\right)$$

$$w_{i+1}=w_i+\frac{1}{6}(k_1+2k_2+2k_3+k_4);\ \ \langle\phi,\zeta\rangle=\int_a^b w(x)\phi(x)\zeta(x)dx$$

matriz Hilbert: $H=\begin{bmatrix}\langle\phi_0,\phi_0\rangle & \cdots & \langle\phi_0,\phi_n\rangle\\ \vdots & & \vdots\\ \langle\phi_n,\phi_0\rangle & \cdots & \langle\phi_n,\phi_n\rangle\end{bmatrix}$; $\ 0=\nabla E(a)=2Ha-2b,\ \ b=[\langle\phi_0,f\rangle\vdots\cdots\vdots\langle\phi_n,f\rangle]'$

Aprox. discreta: $\langle\phi,\zeta\rangle=\sum_{i=1}^{m}w(x_i)\phi(x_i)\zeta(x_i)dx$

$$\phi_0 = 1;\ \ \phi_1 = x - B_1,\ \ B_k = \frac{\langle x\phi_{k-1}, \phi_{k-1}\rangle}{\langle \phi_{k-1}, \phi_{k-1}\rangle};\ \ \phi_k = (x - B_k)\phi_{k-1} - C_k\phi_{k-2};\ C_k = \frac{\langle x\phi_{k-1}, \phi_{k-2}\rangle}{\langle \phi_{k-2}, \phi_{k-2}\rangle}$$

Legendre: $\phi_0 = 1$, $\phi_1 = x$, $\phi_2 = x^2 - (1/3)$, $\phi_3 = x^3 - (3/5)x$, $\phi_4 = x^4 - (6/7)x^2 + (3/35)$, $\phi_5 = x^5 - (10/9)x^3 + (5/21)x$.

Tchebychev: $T_0 = 1$, $T_1 = x$, $T_{n+1} = 2xT_n - T_{n-1}$; $\bar{x}_k = cos\left(\frac{2k-1}{2n}\pi\right)$; $\max_{x\in[-1,1]} |f(x) - P(x)| \leq \frac{1}{2^n(n+1)!} \max_{x\in[-1,1]} |f^{(n+1)}(x)|$

$$\sum_{i=0}^{k} a_i q_{k-i} = p_k,\ k = 0, \ldots, N. \qquad d = -\nabla f(x);\ \ \nabla^2 f(x^k)\, d^k = -\nabla f(x^k).$$

$$a_k = \pi^{-1} \int_{-\pi}^{\pi} f(x) cos(kx) dx,\ k = 0, \ldots, n,\ \ b_k = \pi^{-1} \int_{-\pi}^{\pi} f(x) sin(kx) dx,\ k = 1, \ldots, n-1.$$