Função de Heaviside & Impulso

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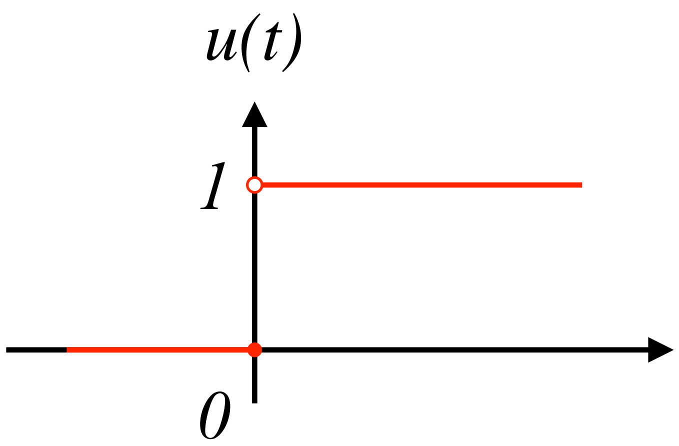
ICMC CeMEAI USP

A função de Heaviside



Função degrau

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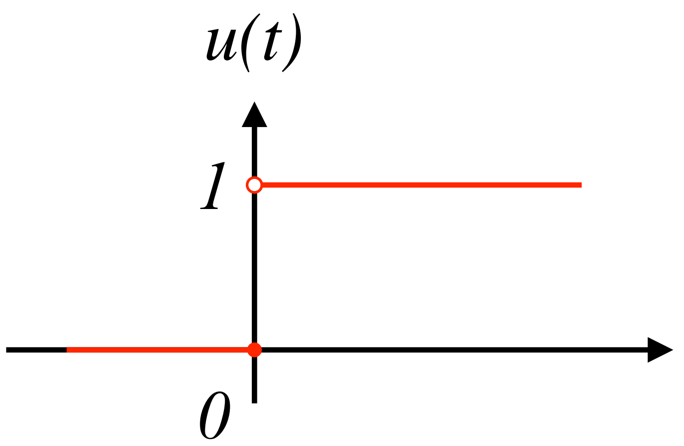
A função de Heaviside



$$u(t) = \begin{cases} 0 & \text{se } t \leq 0 \\ 1 & \text{se } t > 0 \end{cases}$$

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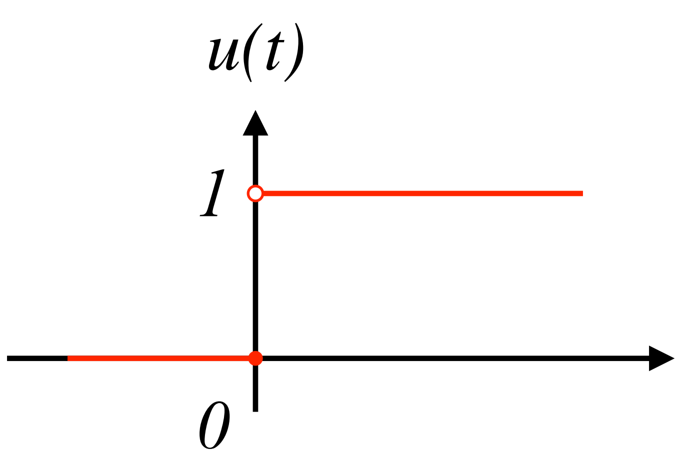
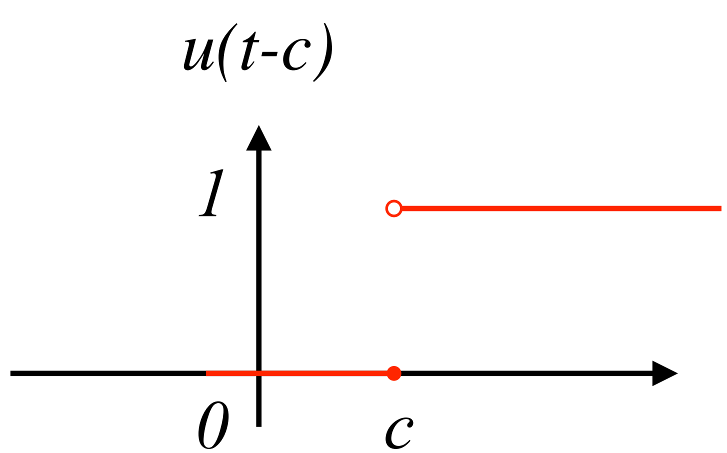
A função de Heaviside

$$u(t-c) \quad \text{é a função } u(t) \text{ translada para o ponto } c$$





$$t - c > 0 \quad \Rightarrow t > c$$

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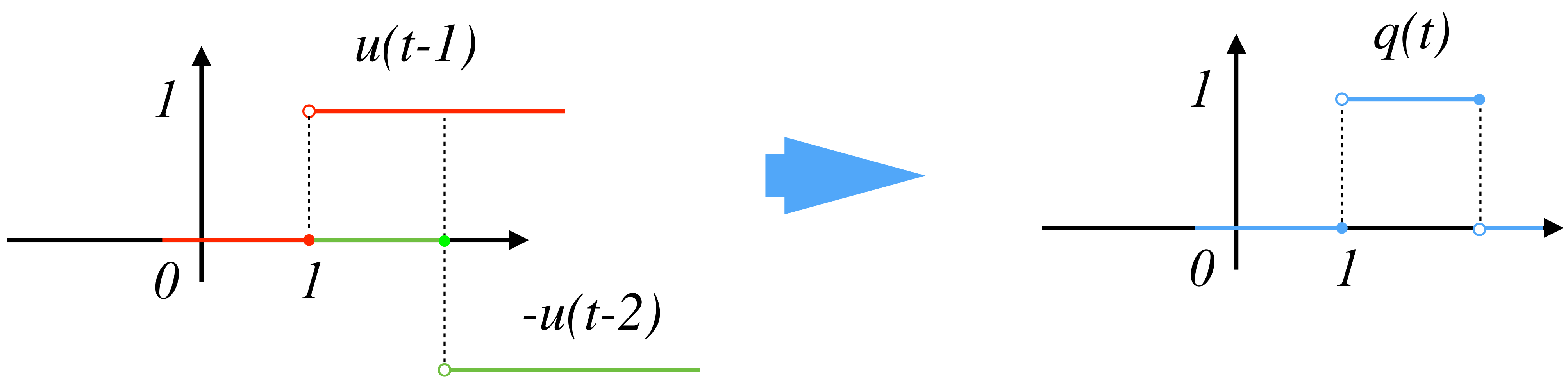
Pulso quadrado



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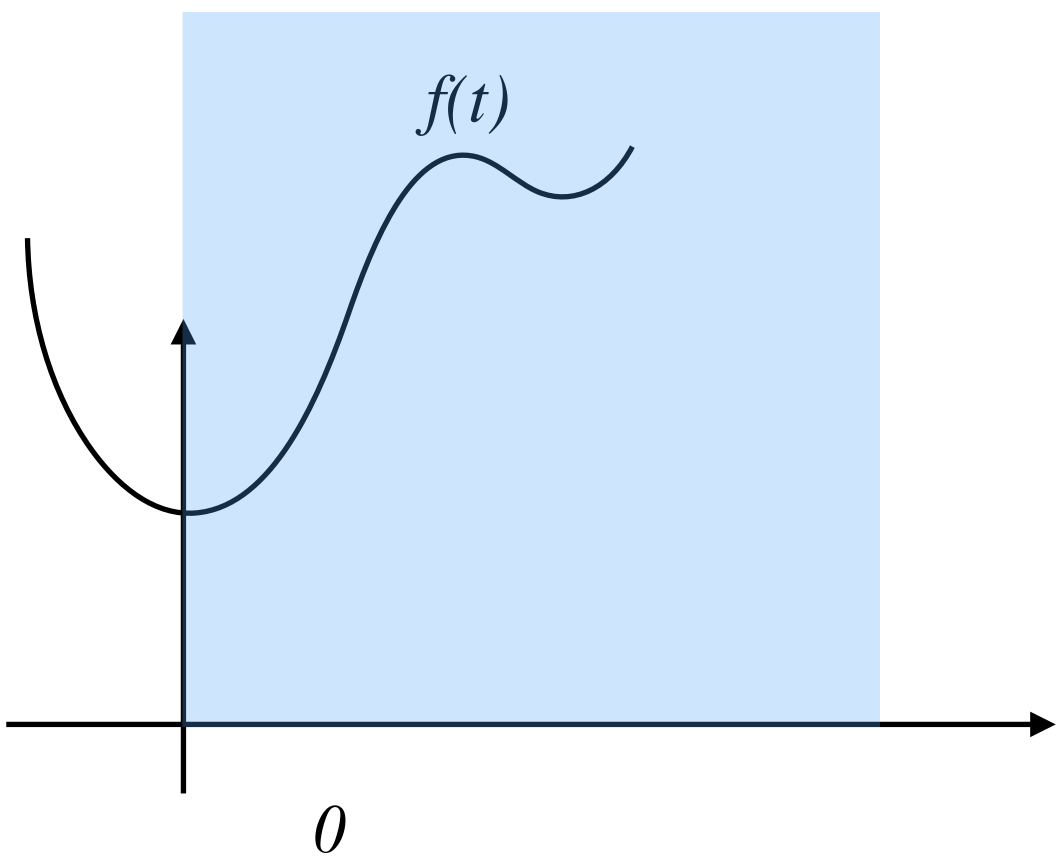
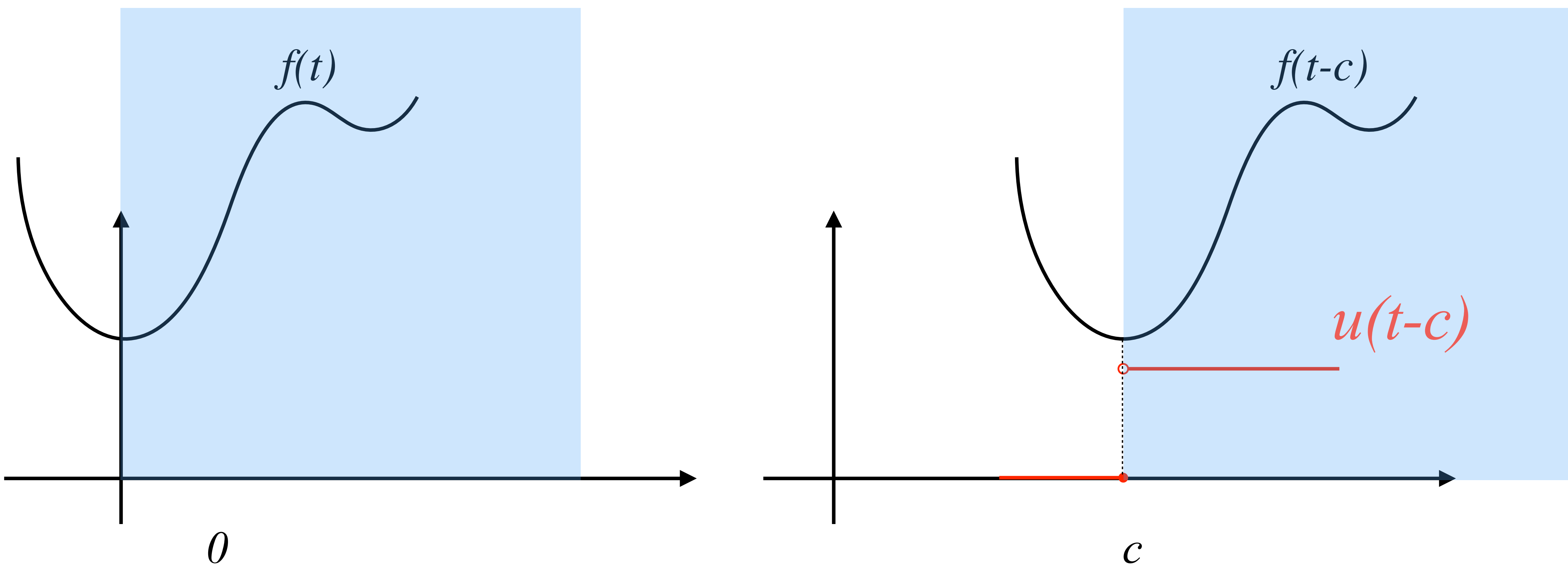
Transformada da Heaviside

$$\begin{aligned}
\mathcal{L}(u(t-c)) &= \int_0^\infty e^{-st}u(t-c)dt \\
&= \underbrace{\int_0^c e^{-st}u(t-c)dt}_{u=0} + \underbrace{\int_c^\infty e^{-st}u(t-c)dt}_{u=1} \\
&= \int_c^\infty e^{-st}dt \\
&= \frac{e^{-sc}}{s}
\end{aligned}$$

A função de Heaviside



A função de Heaviside



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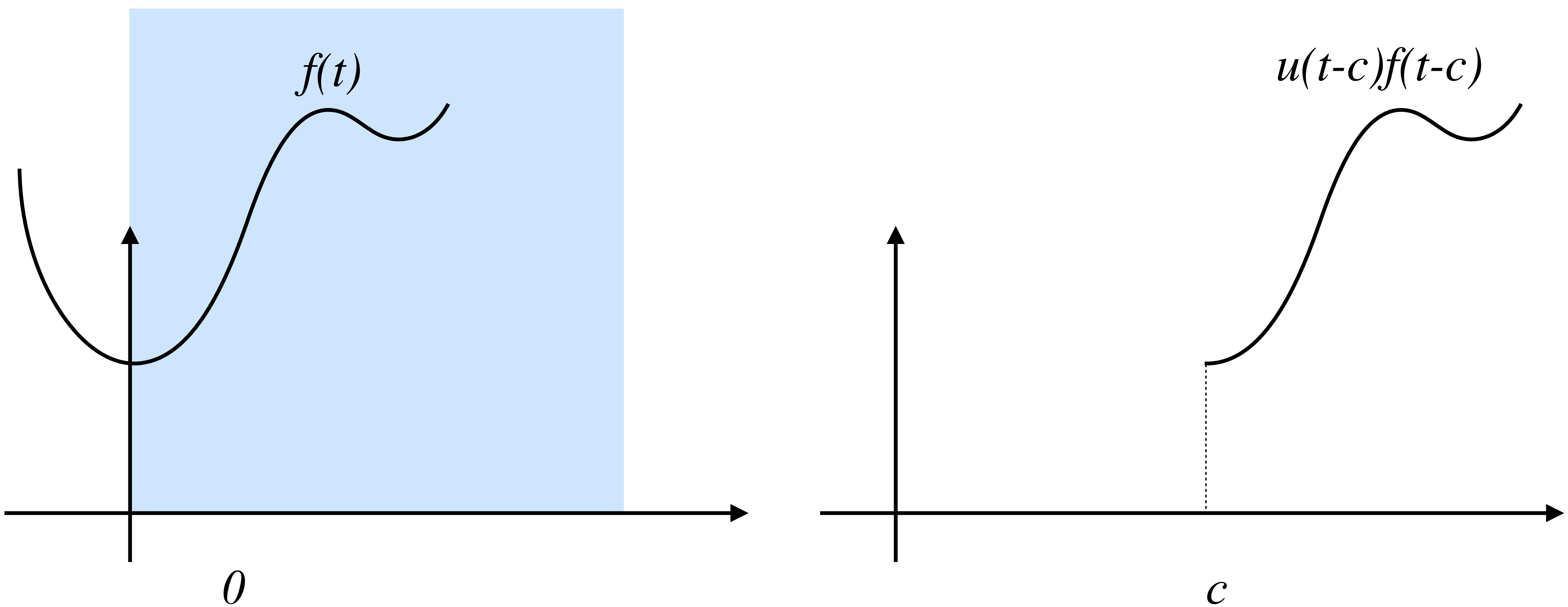
A função de Heaviside



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Translação de uma função

$$g(t) = u(t-c)f(t-c)$$

A função de Heaviside

$$\begin{aligned}\mathcal{L}(u(t-c)f(t-c)) &= \int_0^\infty e^{-st}u(t-c)f(t-c)dt \\ &= \int_c^\infty e^{-st}f(t-c)dt\end{aligned}$$

usando $\tau = t - c$

$$\begin{aligned}\mathcal{L}(u(t-c)f(t-c)) &= e^{-sc}\int_0^\infty e^{-s\tau}u(\tau)f(\tau)d\tau \\ &= e^{-sc}F(s)\end{aligned}$$

Exemplo

Se a transformada for

$$G(s) = \frac{e^{-s}}{s+2} \quad \Rightarrow \quad c = 1 \;\text{ e }\; F(s) = 1/(s+2)$$

Recap

$$\mathcal{L}(u(t-c)f(t-c)) \;=\; e^{-sc}F(s)$$

$$g(t) = u(t-1)e^{-2(t-1)}$$

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Forçamento com pulso quadrado

$$x' + x = q(t)$$

transformando

$$(s+1)X(s) = Q(s) \quad \Rightarrow \quad X(s) = \frac{e^{-s} - e^{-2s}}{s(s+1)}$$

Forçamento com pulso quadrado

$$X(s) = \frac{e^{-s} - e^{-2s}}{s(s+1)}$$

usando que $$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$X(s) = \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-s}}{s+1} + \frac{e^{-2s}}{s+1}$$

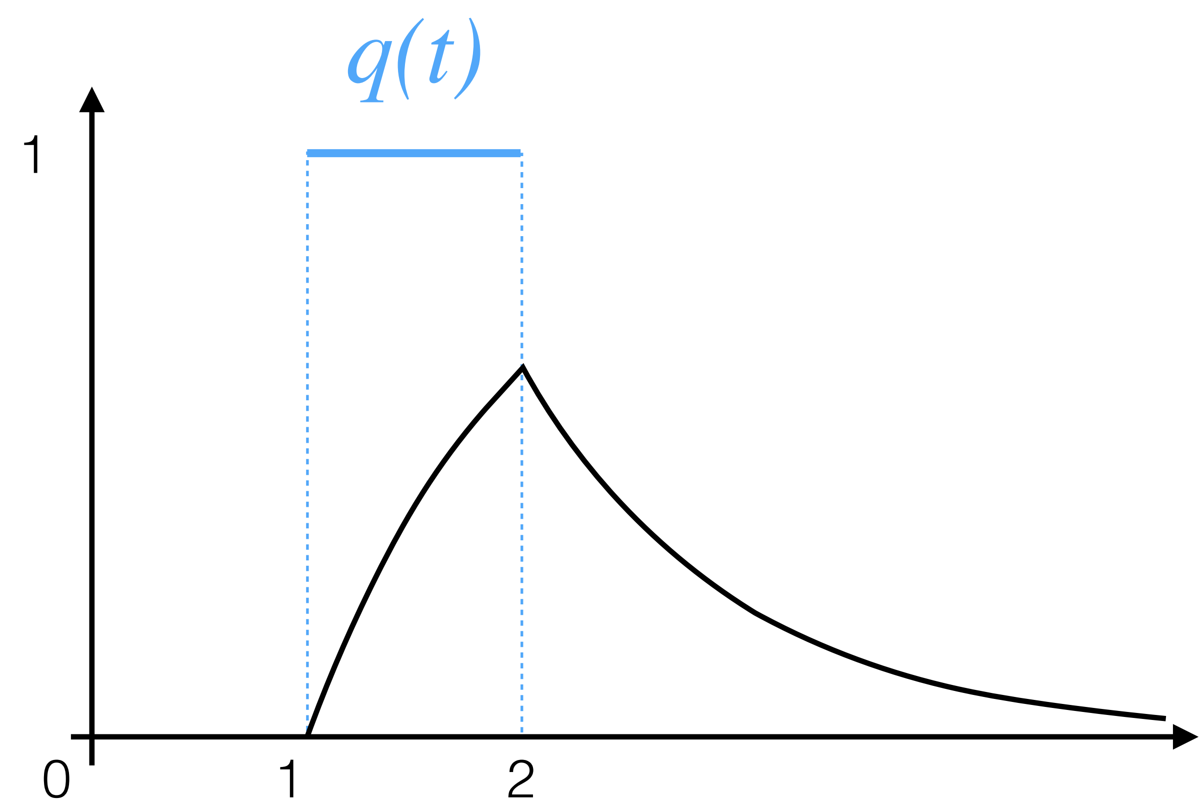
Forçamento com pulso quadrado

$$X(s) = \underbrace{\frac{e^{-s}}{s}}_{u(t-1)} - \underbrace{\frac{e^{-2s}}{s}}_{u(t-2)} - \underbrace{\frac{e^{-s}}{s+1}}_{u(t-1)e^{-(t-1)}} + \underbrace{\frac{e^{-2s}}{s+1}}_{u(t-2)e^{-(t-2)}}$$

Portanto

$$x(t) = u(t-1)\left(1 - e^{-(t-1)}\right) + u(t-2)\left(1 - e^{-(t-2)}\right)$$

Forçamento com pulso quadrado



$$x(t) = u(t-1)\left(1 - e^{-(t-1)}\right) + u(t-2)\left(1 - e^{-(t-2)}\right)$$

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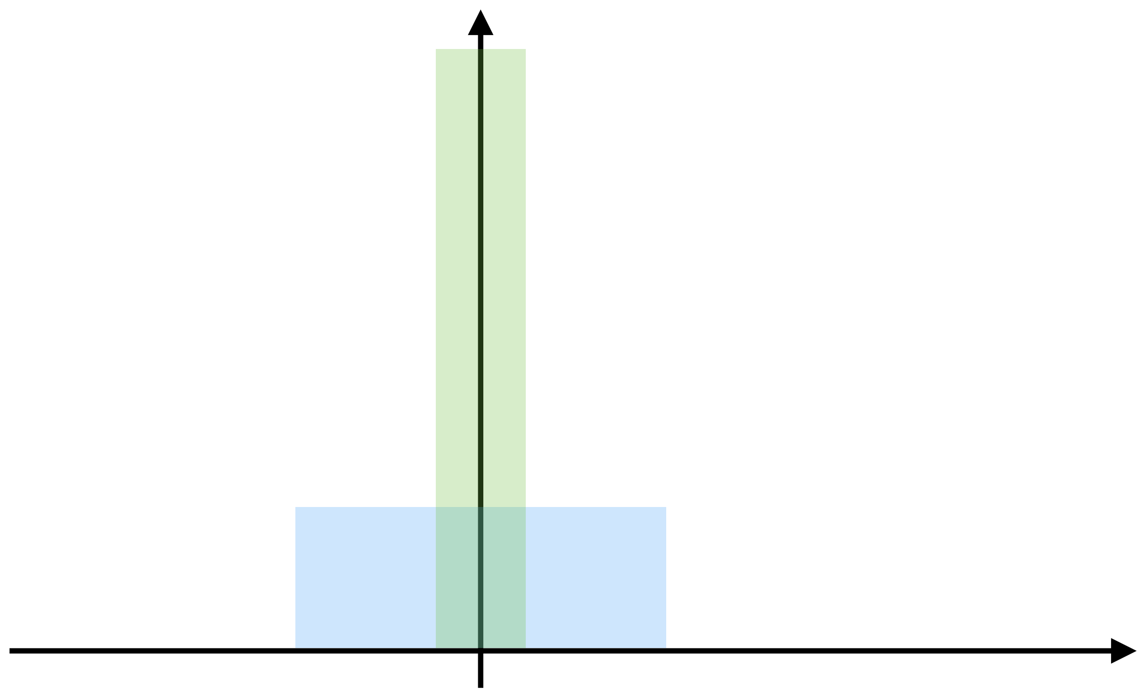
Impulso

$$\int_a^b \delta(t)dt = \begin{cases} 1 & \text{se } 0 \in [a,b] \\ 0 & \text{caso contrario} \end{cases}$$

Impulso

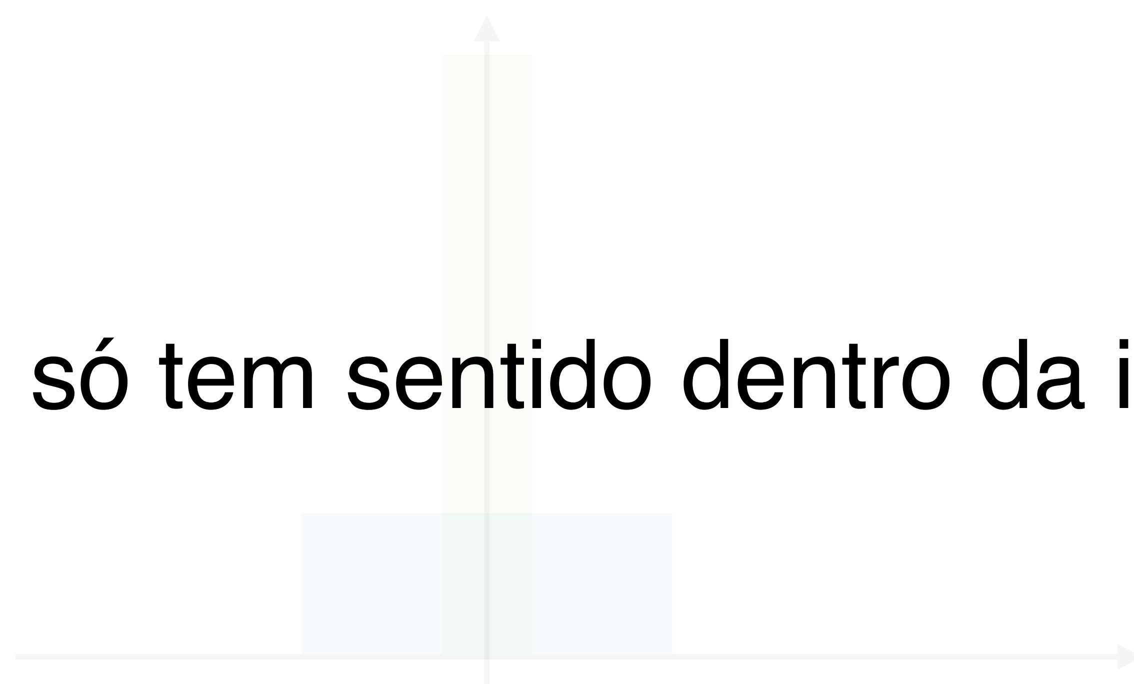
Sequencia de função com mesma area



No limite temos o impulso

Impulso

Sequencia de função com mesma area



$\delta(t)$ só tem sentido dentro da integral

No limite temos o impulso

Impulso

$$\int_{-\infty}^{\infty} \delta(t) f(t) dt = f(0)$$

Impulso

$$\mathcal{L}(\delta(t)) = 1$$

A função de Heaviside e Impulso

$$\frac{du}{dt} \text{ não tem sentido no calculo clássico}$$

A função de Heaviside e Impulso

$$
\begin{aligned}
\mathcal{L}(u'(t)) &= sU(s) - u(0) \\
&= 1
\end{aligned}
$$

A função de Heaviside e Impulso

$$\begin{aligned} \mathcal{L}(u'(t)) &= sU(s) - u(0) \\ &= 1 \\ &= \mathcal{L}(\delta(t)) \end{aligned}$$

A função de Heaviside e Impulso

$$u'(t) = \delta(t)$$