Primeira Lista de Exercícios Simplificada de SMA354 - Cálculo II

Integral definida

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Exercício 1 *Encontrar o valor das integrais definidas:*

(**a**) $\displaystyle\int_{-3}^{2} |x+1|\,dx$ (**b**) $\displaystyle\int_{0}^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$ (**c**) $\displaystyle\int_{7}^{12} dx$ (**d**) $\displaystyle\int_{-2}^{3} \left(5+x-6x^2\right)\,dx$

(**e**) $\displaystyle\int_{1}^{0} t^2\left(t^{\frac{1}{3}}-\sqrt{t}\right)\,dt$ (**f**) $\displaystyle\int_{3}^{2} \frac{x^2-1}{x-1}\,dx$ (**g**) $\displaystyle\int_{0}^{\frac{1}{2}} \frac{1}{\left(1-v^2\right)^2}\,dv$ (**h**) $\displaystyle\int_{0}^{1} x^2 e^x\,dx$

(**i**) $\displaystyle\int_{0}^{\pi/2} e^x\cos(x)\,dx$ (**j**) $\displaystyle\int_{0}^{1} \operatorname{tgh}(x)\,dx$ (**k**) $\displaystyle\int_{1}^{2} x\,2^x\,dx$ (**l**) $\displaystyle\int_{0}^{1} x\,(2x+3)^{99}\,dx$

Exercício 2 *Em cada um dos itens abaixo, encontrar a expressão da função* $f':\mathbb{R}\to\mathbb{R}$*, onde* $f:\mathbb{R}\to\mathbb{R}$*, é dada por:*

(**a**) $f(x)\doteq\displaystyle\int_{0}^{x}\left(t^2+1\right)^{10}dt$ (**b**) $f(x)\doteq\displaystyle\int_{0}^{2}\left(x^3+x^2-7\right)^{5}dx$ (**c**) $f(x)\doteq\displaystyle\int_{x}^{0}\sqrt{u^4+4u^2}\,du$

(**d**) $f(x)\doteq\displaystyle\int_{0}^{x^3}\cos^{\frac{1}{3}}(t)\,dt$ (**e**) $f(x)\doteq\displaystyle\int_{\operatorname{sen}(x)}^{\cos(x)}\sqrt{t^2+1}\,dt$ (**f**) $f(x)\doteq\displaystyle\int_{4x}^{5x}\operatorname{sen}^5(t)\,dt$

Exercício 3 *Estude a paridade das funções que aparecem no integrando das integrais definidas abaixo e depois calcule-as:*

(**a**) $\displaystyle\int_{-1}^{1}\left(x^2+4\right)dx$ (**b**) $\displaystyle\int_{\frac{-17\pi}{4}}^{\frac{17\pi}{4}}\left[\operatorname{sen}\left(x^3\right)-x^7\cos(x)\right]dx$ (**c**) $\displaystyle\int_{-1}^{1}\frac{x^3}{x^2+1}\,dx$

Exercício 4 *Verifique que*

(**a**) $$\int_{-\pi}^{\pi}\cos(k\,x)\cos(m\,x)\,dx=\begin{cases}0, & \textit{para } \underline{k},\ \underline{m} \textit{ são inteiros positivos e } k\neq m,\\ \pi, & \textit{para } \underline{k},\ \underline{m} \textit{ são naturais e } k=m,\\ 2\pi, & \textit{para } k=m=0\end{cases}.$$

(**b**) $$\int_{-\pi}^{\pi}\operatorname{sen}(k\,x)\,\operatorname{sen}(m\,x)\,dx=\begin{cases}0, & \textit{para } \underline{k},\ \underline{m} \textit{ são naturais e } k\neq m,\\ \pi, & \textit{para } \underline{k},\ \underline{m} \textit{ são naturais e } k=m\end{cases}.$$

(**c**) $\displaystyle\int_{-\pi}^{\pi}\cos(k\,x)\,\operatorname{sen}(m\,x)\,dx=0$, *para* $\underline{k}$ *é inteiro positivo e* $\underline{m}$ *é natural.*

Exercício 5 *Suponha a função* $f:[-2,0]\to\mathbb{R}$ *é contínua em* $[-2,0]$ *e que* $\displaystyle\int_{-2}^{0}f(x)\,dx=3$. *Calcule* $\displaystyle\int_{0}^{2}f(x-2)\,dx$.

Exercício 6 *Suponha a função* $f:[-1,1]\to\mathbb{R}$ *é contínua em* $[-1,1]$ *e que* $\displaystyle\int_{-1}^{1}f(t)\,dt=5$. *Calcule* $\displaystyle\int_{0}^{1}f(2x-1)\,dx$.